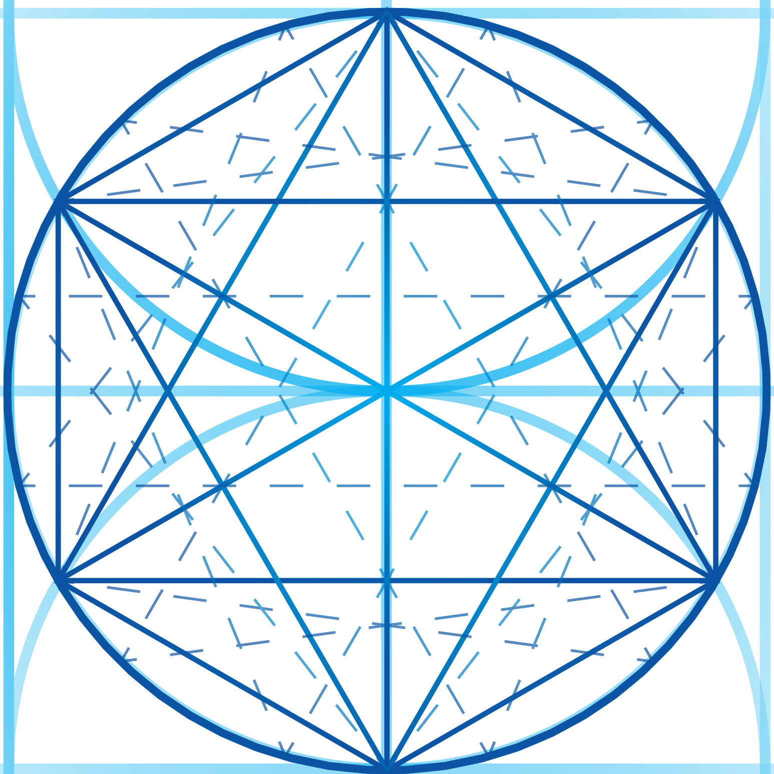
THE BEST OF

MATHMAG

1992 - 1996



WITH ANSWERS

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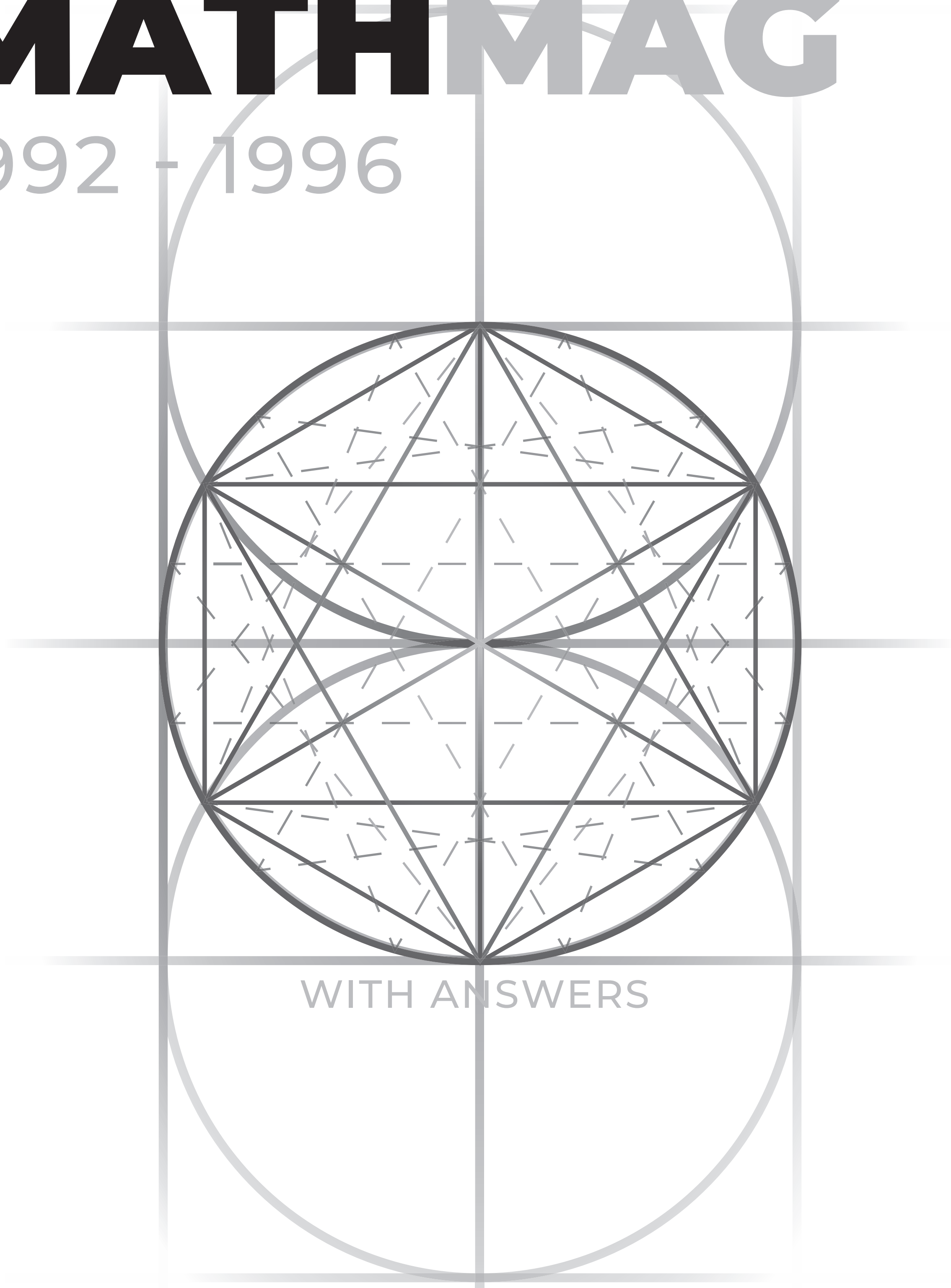
THE BEST OF MATHMAG 1992 - 1996



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The Mathematical Association of Western Australia Inc.

MAΣWA

Contents

Acknowledgements

Mathmag is produced by the Mathematical Association of Western Australia for the enjoyment of middle to upper primary students across Australia.

Numerous requests have been received for back issues of Mathmag and answers to the various activities. The Best of Mathmag 1992 - 1996 is an attempt to put together the most popular activities for the five year period from 1992 - 1996.

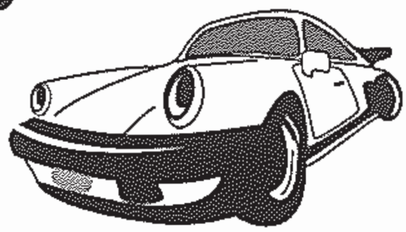
Credit must be given to Alistair McIntosh and Joy Scott who produced the original Mathmag for Western Australian children.

Note: at the top of each page there is a letter. These correspond to:

- M: Measurement,
- S: Geometry (Space), and
- N: Number.

Editor: Paul Swan
Typesetting: J. Swan, L. Swan and D. Swan

How Fast Do You React?



- Cover the letters below.
- The idea behind this test is to touch each letter in alphabetical order.
- Ask a friend to time you. When your friend says go, uncover the chart and start.

D	C	J	L
F	K	H	I
A	G	B	E

- Record the time taken to complete the reaction test.
- Five seconds is excellent, seven seconds good, whilst nine seconds is average. Above nine seconds go back to bed.
- Survey your class to find out the reaction times of each class member. Find the average reaction time. Compare this to the fastest and slowest reaction times recorded in your class.
- Try making your own test chart by jumbling up various letters of the alphabet.

Measure Matters I

REACTION DISTANCE

To find the ability of another person to react try the following experiment. One person holds a ruler with the zero mark facing downwards and lined up with their partner's open hand. The thumb and first finger should be about 8 cm apart.

The ruler is dropped without warning and the other person tries to catch the ruler. The reaction ability of the person is read off the ruler.

HANDY MATHS

Place your hand over a piece of graph paper and trace around it. Count the squares to determine the area of your hand. When counting the squares ignore half squares and those less than half. Count all others as full squares.

It takes about 100 handprints to cover your entire body. Find the surface area of your body.

Does it matter whether your fingers were closed or spread?

Find the perimeter of your hand when it is open and when it is closed, You may wish to use string to help you.

Does it matter whether your fingers were closed or spread?

Measure Matters II

How much skin do you think it takes to cover the sole of your foot?

Trace the outline of your foot onto a piece of graph paper and count the number of squares to determine the area of the sole of your foot.

Compare your answer to your original estimate.

How much skin do you think it takes to cover your entire body?

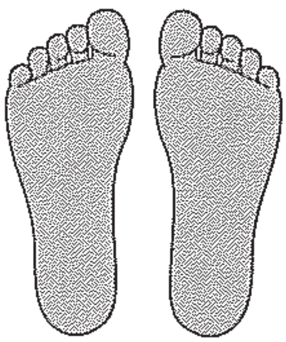
A simple rule of thumb that can be used to find the area of your entire body is given below.

AREA OF SOLE X 100 = TOTAL SKIN AREA

You should find that your answer lies between one and two square metres.

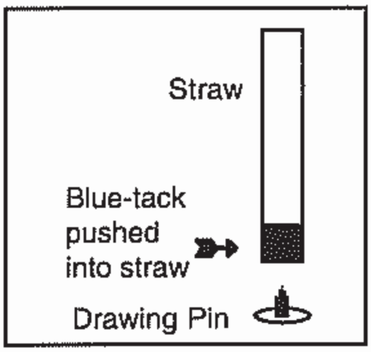
Remember 10 000 cm^2 = 1 m^2

Compare your skin area to that of an adult.

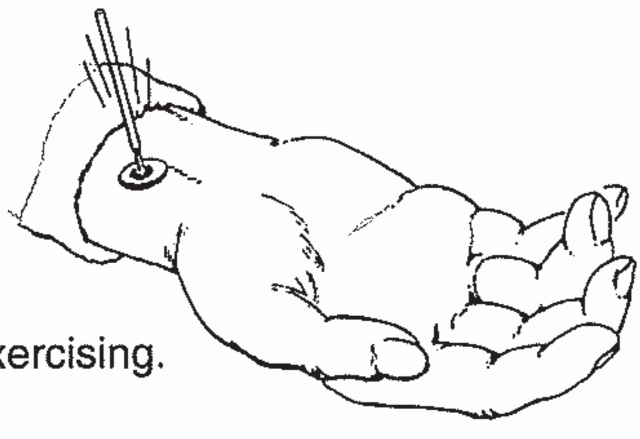


Heart Beat

- The average person's heart beats 60-75 times per minute.
- How many times per day, week, year, lifetime is this?
- You can measure your own heartbeat by taking your pulse. To do this you will need a straw, a drawing pin and some blu-tack or plasticine.



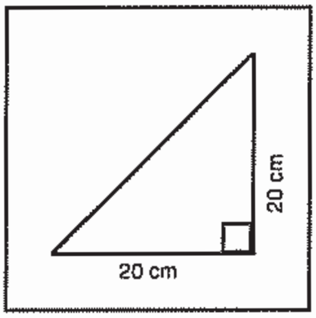
- Cut the straw in half and push some blu-tack into the end. Stick the drawing pin into the end of the straw containing the blu-tack.
- Lay your hand, palm up on your desk, and then place your pulse measurer on your wrist. Move it around until you find your pulse. (The straw will move from side to side.) Count how many times your heart beats in one minute by counting the movements of the straw. Record this and then exercise for five minutes. (e.g. Jog up and down on the spot, do some step ups etc.) Count how many times your heart beats in one minute immediately after exercising.

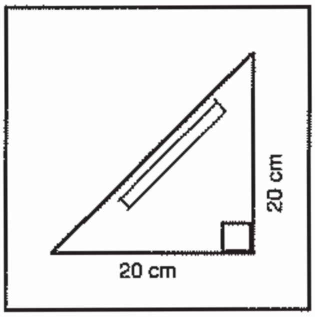


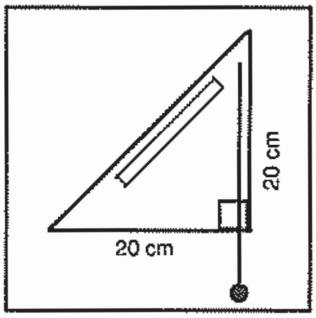
TOO TALL TO MEASURE

To measure the height of a tree or flagpole you will need to make a **Clinometer**. You will require some stiff cardboard, a straw, sticky tape, string and a weight.

Cut out a piece of cardboard in the shape of a right isosceles triangle 20 cm by 20 cm.





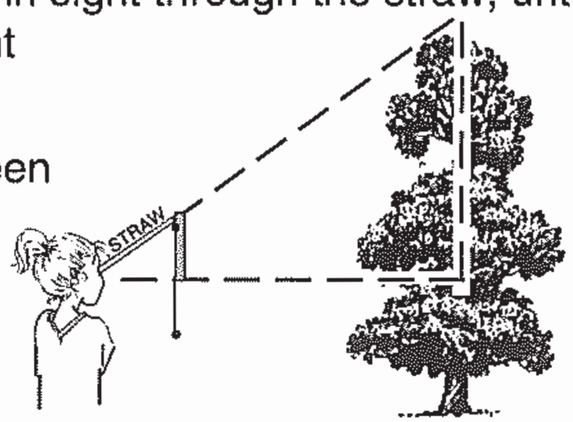


Attach the straw along the longest side of the triangle using the sticky tape. Tie a weight on to the end of the piece of string, attach the other end to the card to form a plumb line.

Your clinometer is now complete.

To measure the height of a tree, look at the top of the tree through the straw. Walk either towards or away from the tree, whilst keeping the tree in sight through the straw, until the plumb line hangs straight down.

Measure the distance between yourself and the tree, then add on your own height and you will have found the height of the tree.



FINDING SQUARES

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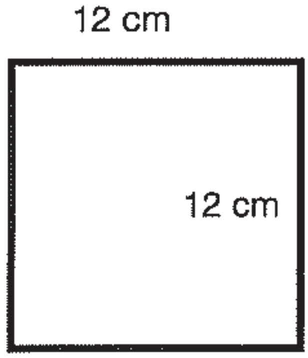
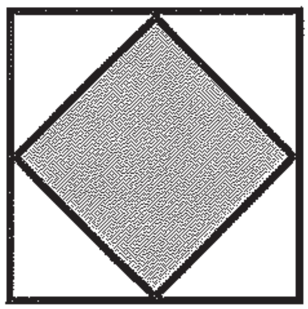
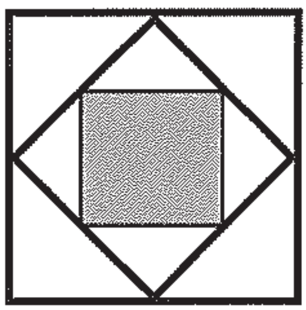
- Cut out a piece of paper to form a 12 cm x 12 cm square. (10 mm graph paper would be ideal.) Find the area of the square.



- Fold the four corners into the centre of the piece of paper so that a new square is formed. You may like to unfold the paper and cut the corners off. Determine the area of this new square.



- Fold the corners again. Find the area of the new square.
- Repeat once more, noting the area.



- Compare the areas of the various squares which were formed. Discuss what you find with your neighbour.
- What do you think would happen if you repeated the process once again?

Try it and see.

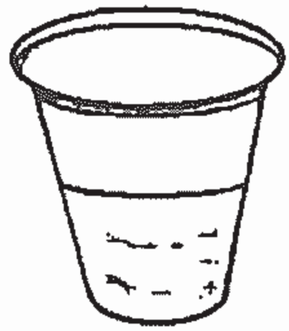
HOROLOGY

The Marking of Time

M

WATER CLOCK

To make a Water clock you will require a styrofoam (or similar) cup and a plastic cool drink container.



- Cut the top off a plastic cool drink container so that a plastic cup fits snugly inside.
- Pierce a small hole through the bottom of the cup.
- Fill the cup with water and allow to drip into the container.
- Using a watch or clock mark the height of the water after certain time intervals. (e.g. perhaps every minute.)
- To measure larger time periods you can use two plastic cool drink bottles and allow the contents of one to drip into the other (similar to the sand timer).

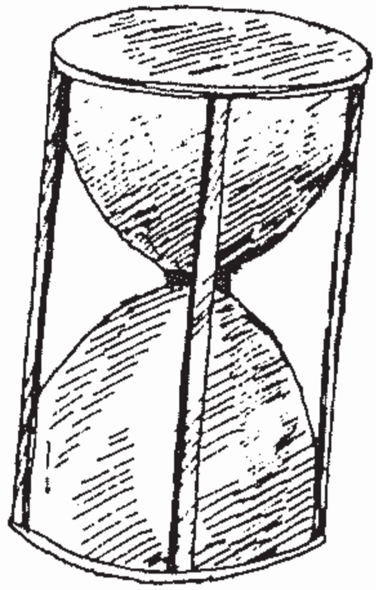
M

HOROLOGY

The Marking of Time

HOUR GLASS

Many people have egg timers which are basically sand clocks.



You can make your own sand timer using two plastic cool drink bottles, some cardboard, tape and some fine sand.

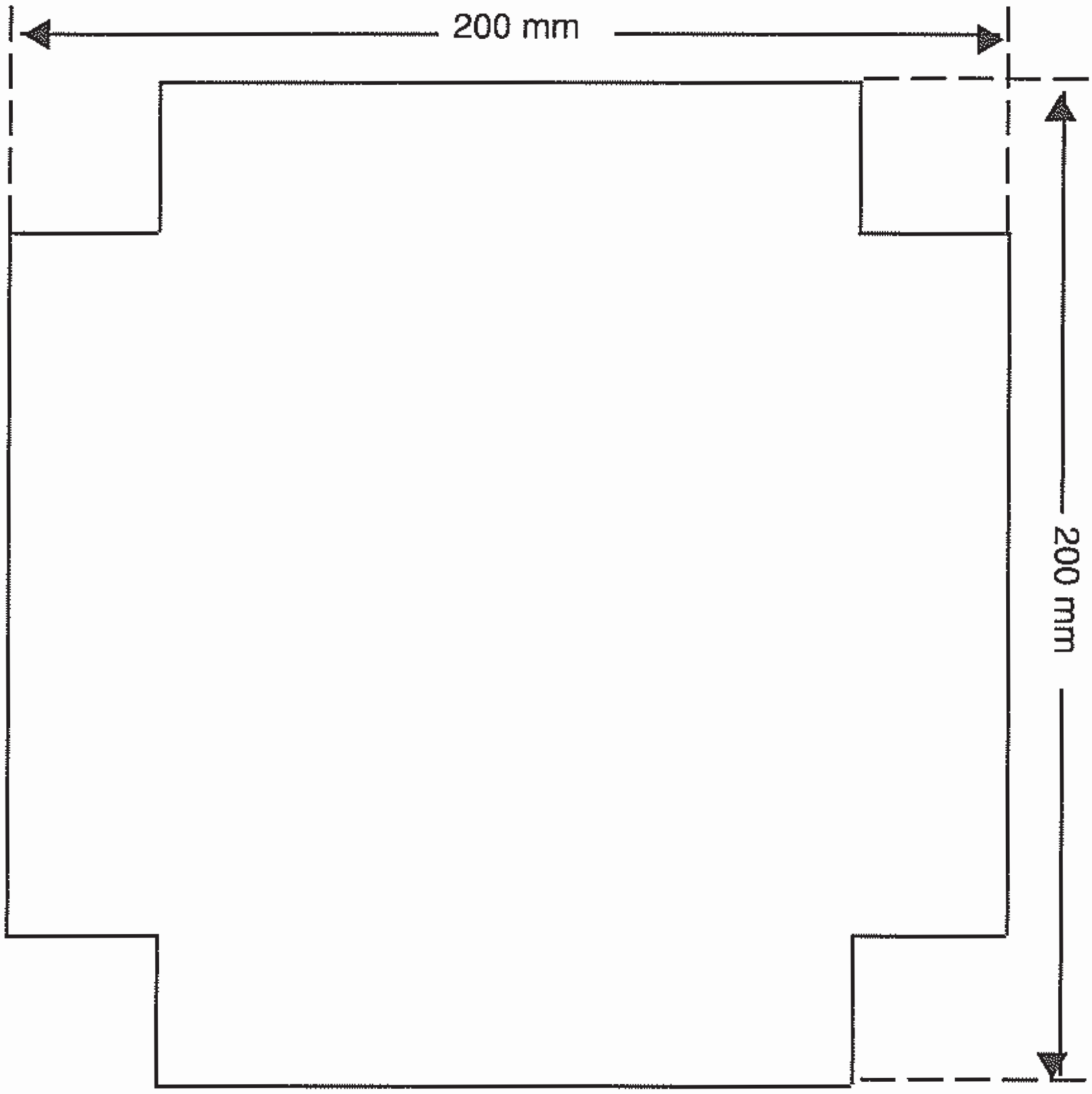
Ensure that the sand and the bottles are dry before proceeding.

- Trace a circle around the rim of the cool drink container on to the cardboard. Cut it out.
- Punch a small hole in the centre.
- Fill one container with sand.
- Tape the cardboard circle on to the container
- Place the empty container on the top and tape the two containers together.
- Turn your timer over and, using a stopwatch, measure the amount of time that it takes the sand to run from one container into the other. Adjust the size of the hole or the amount of sand until the exact amount of time you wish is achieved.

SQUARE TRAYS

M

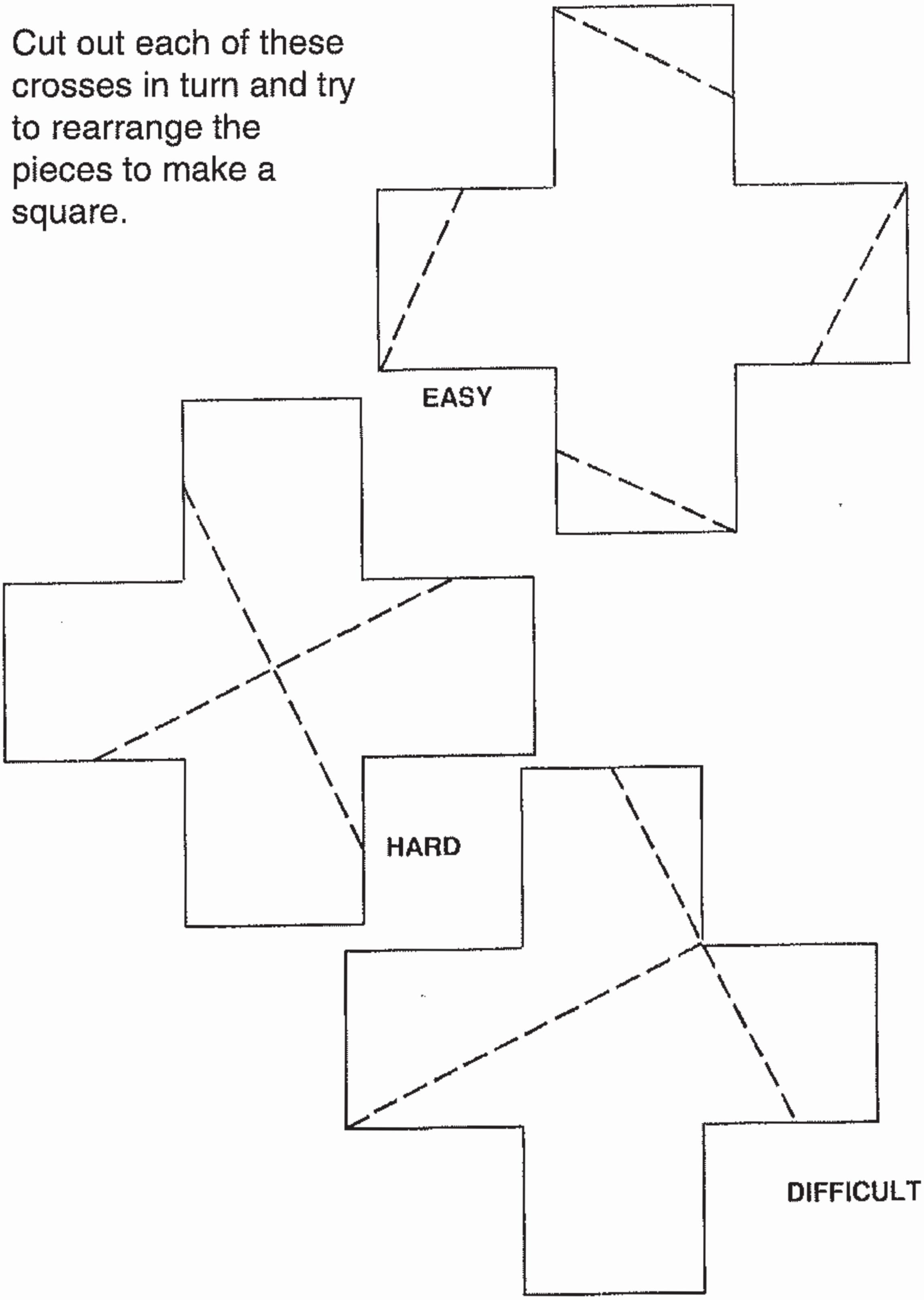
A tray is to be made by cutting a square corner from a 200 mm x 200 mm piece of card.



The sides are then folded and glued to produce the tray. Experiment with different sized corner cuts to produce a tray which holds the greatest amount. You will need a pair of scissors, and some 10 mm graph paper. Restrict your corner cuts to 10 mm, 20 mm, 30 mm etc.

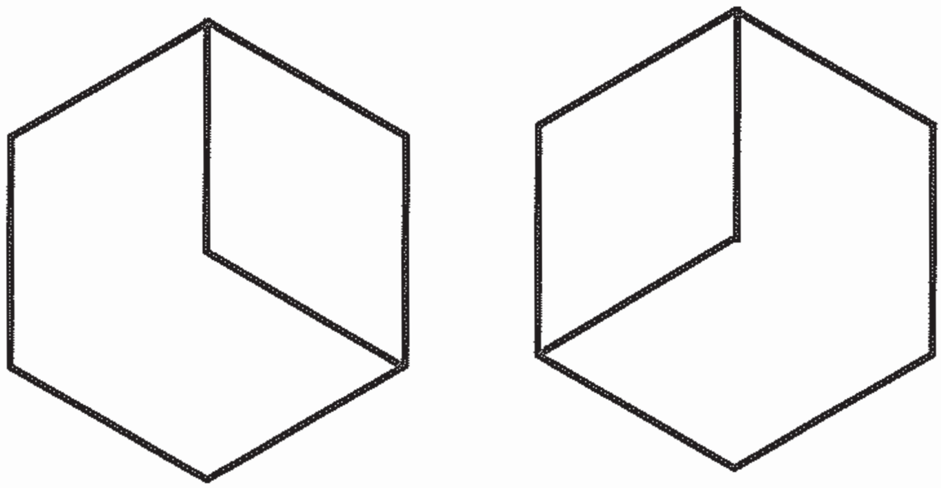
Squares from Crosses

Cut out each of these crosses in turn and try to rearrange the pieces to make a square.



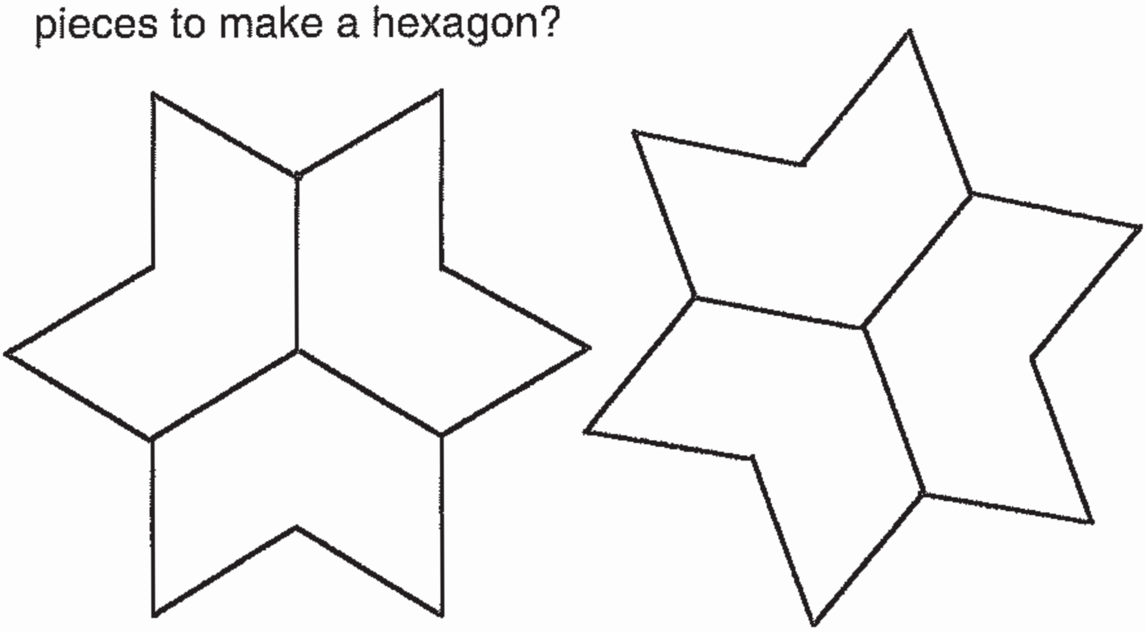
HEXAGONS INTO STAR

Cut out the two hexgons into four pieces as shown. Can you rearrange them to make a 6 pointed star?



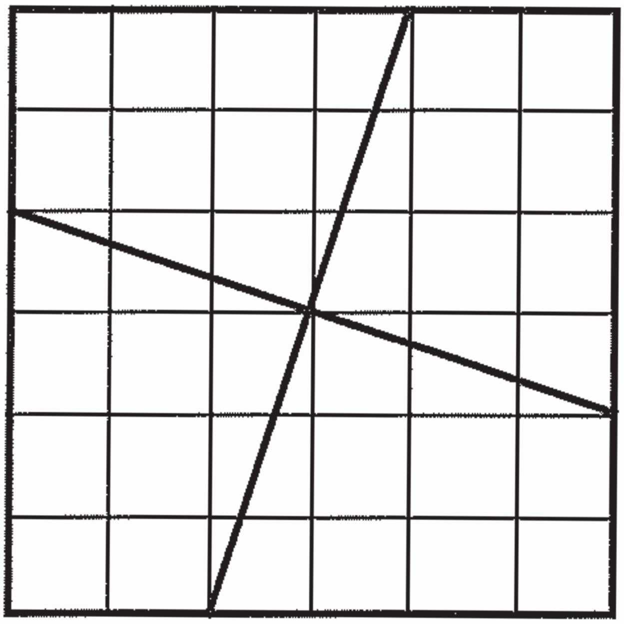
STARS INTO HEXAGON

Now cut out these two stars. Can you rearange the six pieces to make a hexagon?

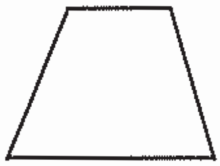


CUTTING AND SHAPING

S

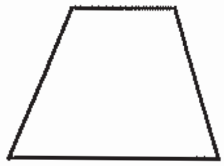


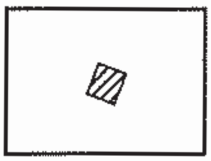
- ✂ Cut out a 6 cm x 6 cm square.
- ✂ Rule two lines as shown in diagram.
- ✂ Cut along the lines.
- ✂ What do you notice about the four pieces?
- ✂ Try fitting the four pieces together to form:



A Trapezium.

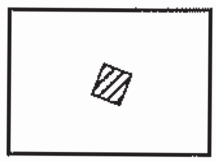
(You may need to flip 2 pieces)

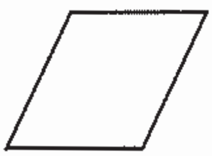




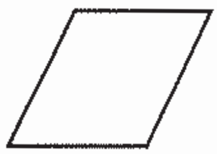
A Rectangle with a rectangular hole.

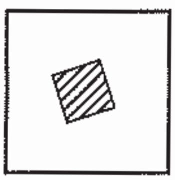
(You may need to flip 2 pieces)



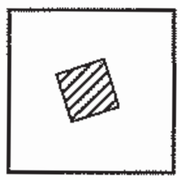


A Rhombus.



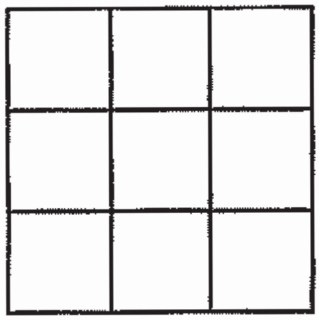


A Square with a square hole.

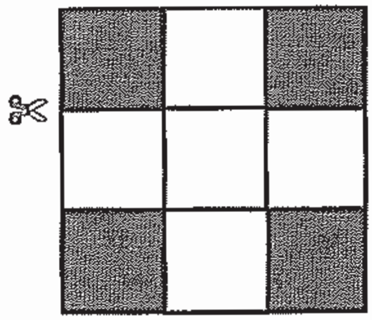


S

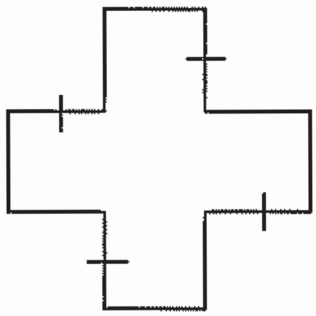
CROSS CUT



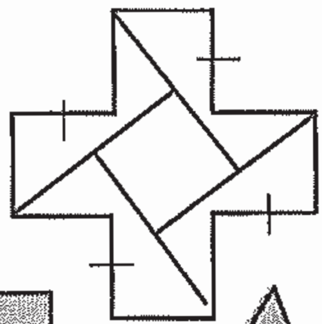
- Fold a square piece of paper in thirds.
- Crease and then open up.
- Rotate the piece of paper 90° and fold in thirds again. Crease and open up. Your piece of paper should now be divided into ninths.



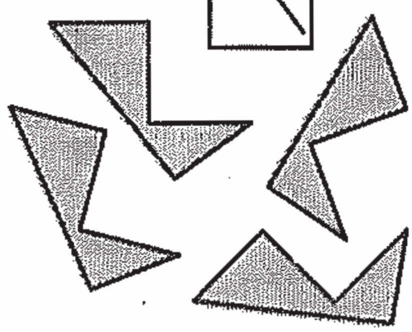
- Cut away the four corner squares so that a cross is formed.



- Mark in the middle of the edges shown and join these points to the opposite corners.
- Cut along the lines so that four shapes that are the same are formed.



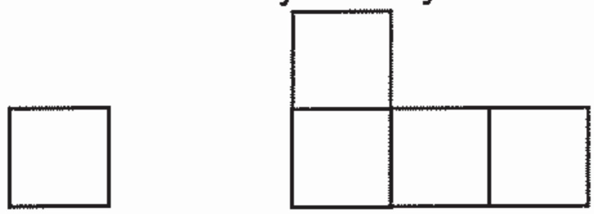
- Throw away the square that was in the middle.
- Try to form a square with the remaining four pieces.



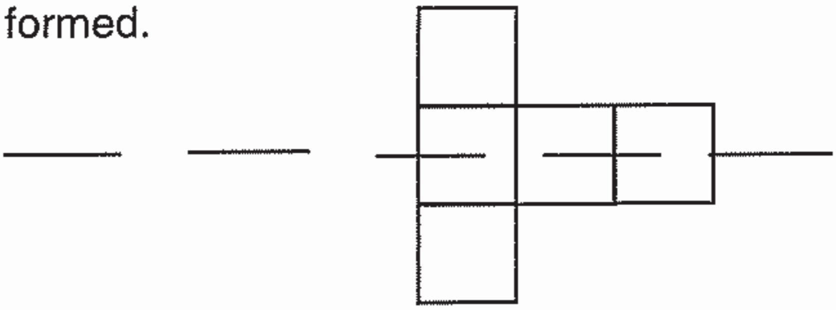
- Try to form another square with an open cross in the middle.

Symmetry

❉ Make a copy of the two shapes shown below and then put them together to produce a new shape with either line or rotational symmetry or both.



❉ For example, by placing the single square on one end of the 'L' shape a 'T' shape with line symmetry is formed.



A line of symmetry runs through the middle of the 'T'. *How many others can you make?*

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

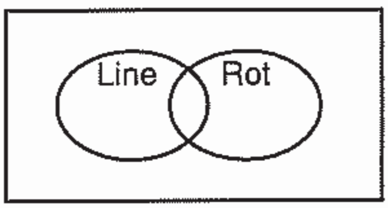
❉ Which letters of the alphabet have:

Line symmetry.

Rotational symmetry.

Both.

You may like to use a Venn diagram to classify the letters

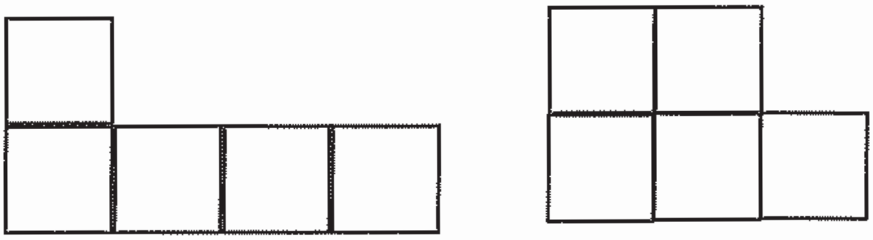


❉ Look through a newspaper and cut out examples of logos with either line symmetry, rotational symmetry or both.

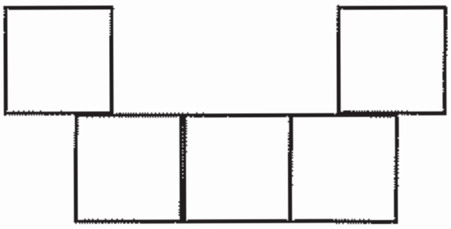
PENTOMINOES

S

A pentomino consists of five squares connected together along their sides like this

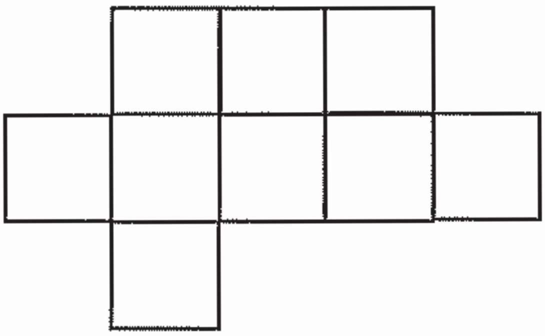


But not like this



There are twelve pentominoes in all. Try to find them all. You may like to cut out some tiles to help you find them. Record your answers on graph paper.

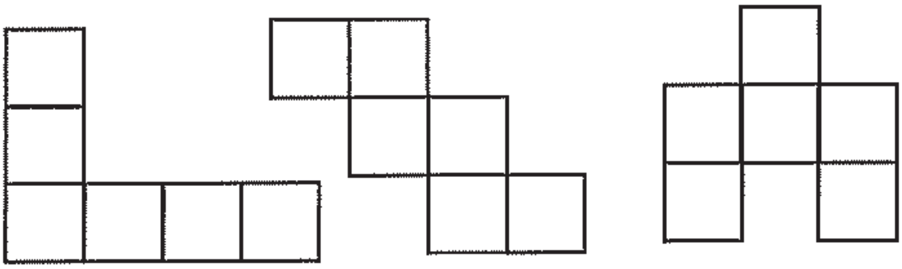
Try to fit all of your pentominoes over the shape shown here.



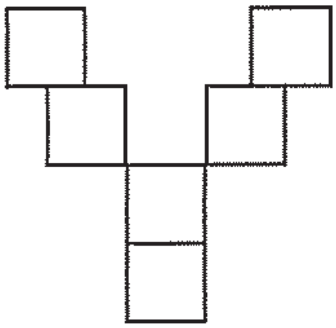
Which pentominoes can be folded to form boxes without lids? (You may like to cut out your pentominoes to check your answers.)

HEXOMINOES

Hexominoes are shapes made with six squares connected together along their sides like this

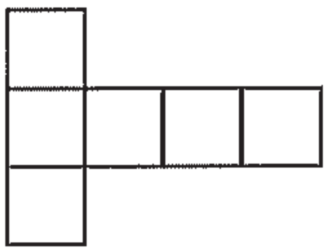


But not like this



There are 35 different Hexominoes.
Try working with a partner to find them all. You may like to cut out some square tiles to help you find them. Remember to record your answers.

The 'T' shape Hexominoes shown on the right may be folded to make a cube.



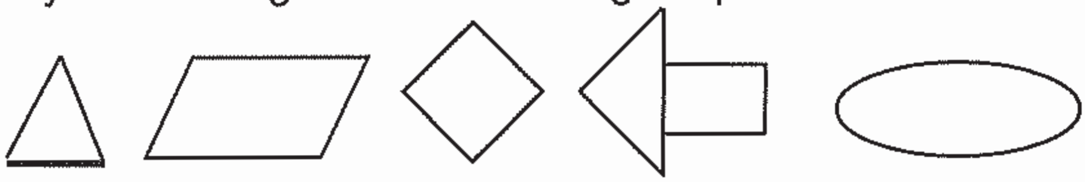
Altogether there are 11 Hexominoes that may be folded to form a cube. Choose the 11 that you think will form cubes and cut them out to check.

TESSELLATIONS

S

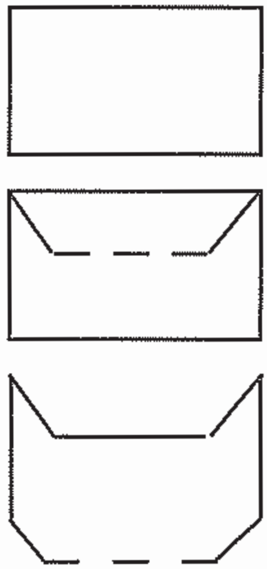
A shape will tessellate if a tiling pattern can be produced without using any other shapes and without there being any gaps between shapes.

Try tessellating with the following shapes.

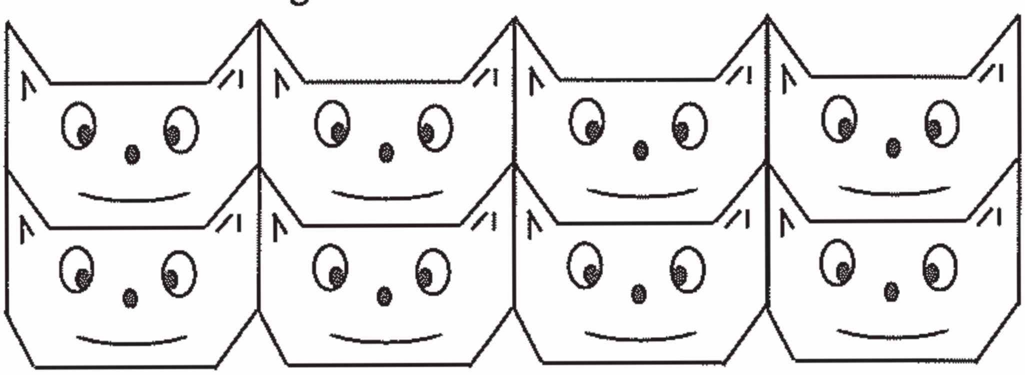


You will notice that not all shapes tessellate.

To produce some interesting tessellations use the following procedure.



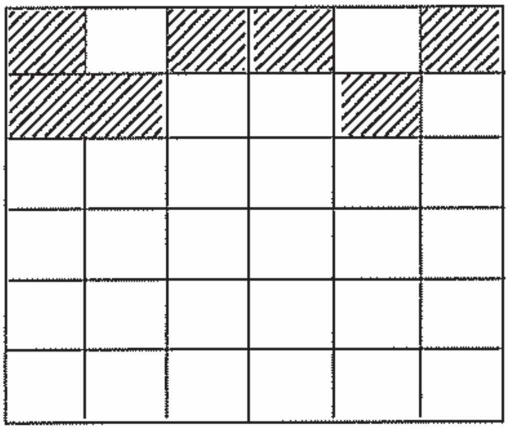
- Start with a shape you know tessellates.
- Cut one piece out and stick it on to the opposite side.
- This new shape will also tessellate.
- Add some artwork to create an interesting effect.



N

COIN TOSS

A coin was tossed and the results placed on a grid. Every time a head came up a square was shaded. If a tail turned up the square was left blank. The grid on the right shows the results for the 12 tosses. H T H H T H H H T T H T .



Draw up two 6 x 6 grids. Shade one grid to indicate what you think might happen if a coin is tossed 36 times. Toss a coin 36 times and record the results on the second grid as you proceed.

- *Were you surprised by the pattern that was formed?*
- *Did you expect the pattern to be ordered and uniform?*

Compare your predictions to your result.

- *Note what happened after you had a run of three head or tails. Is that what you might expect?*
- *Compare your results to those of your friends. In what ways were they the same? Different?*

Try the experiment again, but this time try rolling the coin instead of tossing it. Remember to predict your results before trying the experiment.

What do you think would happen if you rolled a die and recorded odd numbers by shading and even numbers by leaving a blank? Try it. Record your results on a 6 x 6 grid.

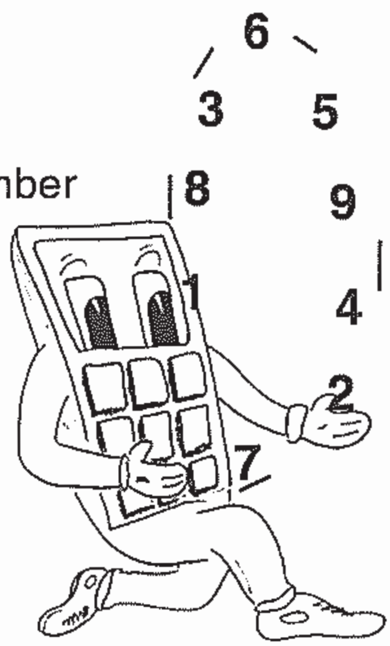
MATHΣMATICAL TRICKS I

N

Σ	Choose any two consecutive numbers.	
		e.g **6 and 7**
Σ	Add them	**13**
Σ	Add 9	**22**
Σ	Divide by 2	**11**
Σ	Take away your first number	**?**

Try with other numbers.

What do you notice?



Σ	Choose any 2 digit number.	e.g. **62**
Σ	Multiply by 7	**434**
Σ	Multiply by 1443	**??????**

What happens?

Try with other numbers.

Why do you think this happens?

You might like to try these tricks on your Mum and Dad.

MATHΣMATICAL TRICKS II

Σ	Choose any two numbers less than 10.	**e.g 3 and 7.**
Σ	Multiply the first number by 5.	**15**
Σ	Add 7.	**22**
Σ	Multiply by 2.	**44**
Σ	Add the second number.	**51**
Σ	Take away 14	**37**

The answer contains your two numbers in order.

Try with other numbers.

To guess a person's age

Σ	Ask them to write down a three digit number where all three digits are different	**165**
Σ	Rearrange the digits to create a different number	**615**
Σ	Subtract the smaller number from the larger	**615** **– 165** **= 450**
Σ	Ask the person to add their age and tell you the answer	**450** **+ 13** **= 463**

To work out their age add the digits of the number they give you, i.e. 4 + 6 + 3 =13,   1 + 3 = 4 until you reach a single digit number.

Once you have done this, add 9s until you reach a figure you consider is close to the person's age (e.g. 9 + 4 = 13, 13 + 9 = 22, 22 + 9 = 31 and so on).

MATHΣMATICAL TRICKS III

N

Try this trick with a friend

Use the steps below to find a person's **age** and the **month** in which they were born.

Give your friend a calculator and ask your friend to:

Σ Enter the number of the month In which they were born (Jan. = 1, Feb. = 2, Mar = 3 and so on.

Σ Double it.

Σ Add 5.

Σ Multiply by 50.

Σ Add their age.

Σ Subtract 365 for the number of days in a year.

Σ Ask your friend to pass the calculator to you.

Σ Add 115 to reveal your friend's birth month and age.

Σ You might like to try this trick on your teacher or Mum and Dad!

MATHΣMATICAL TRICKS IV

N

Try this trick with a friend.

Use the steps below to find someone's **house number and age.**

Give your friend a calculator and ask your friend to:

- Σ Double their house number.
- Σ Add 5.
- Σ Multiply by 50.
- Σ Add their age.
- Σ Add 365 for the number of days in a year.

- Σ Ask your friend to pass the calculator to you.
- Σ Subtract 615 and you will reveal your friend's house number followed by their age.

You might like to try this trick on your teacher or Mum and Dad!

Table Square Sums

N

Consider the following 6 x 6 table square:

1	2	3	4	5	6
2	4	6	8	10	12
3	6	9	12	15	18
4	8	12	16	20	24
5	10	15	20	25	30
6	12	18	24	30	36

What is the sum of the numbers in the top row?

.... the second row?

.... the fourth row?

.... the bottom row?

What is the sum of **all** the numbers in the square?

What about a 4 x 4,

an 8 x 8,

a 10 x 10 square

The Ninety Nine Times Table

N

Complete the nine times table

1	x	9	=	6	x	9	= ....
2	x	9	=	7	x	9	= ....
3	x	9	=	8	x	9	= ....
4	x	9	=	9	x	9	= ....
5	x	9	=				

✎ Write about any patterns you notice.

✎ Add the digits which form the answer. *What do you notice?*

✎ Try to write a rule to determine whether a number is evenly divisible by nine.

✎ Does the pattern continue for larger multiples of nine, e.g. 34 x 9?

Now complete the ninety-nine times table.

1	x	99	=	6	x	99	= ....
2	x	99	=	7	x	99	= ....
3	x	99	=	8	x	99	= ....
4	x	99	=	9	x	99	= ....
5	x	99	=				

✎ Do similar patterns exist in the ninety-nine times table as in the nine times table?

✎ Add the digits which form the answer. *What do you notice?*

✎ *What other patterns do you notice?*

Tables Test: 8 and 9

N

Test your knowledge of the eight and nine times tables on these questions.

1)
```
  123456789
x         9
 ..................
+        10
= ..................
```

2)
```
  12345678
x        9
 ..................
+        9
= ..................
```

3)
```
  123456789
x         8
 ..................
+         9
= ..................
```

4)
```
  12345678
x        8
 ..................
+        8
= ..................
```

5)

9 x 123456789 + 10 =
9 x 12345678 + 9 =
9 x 1234567 + 8 =
9 x 123456 + 7 =
9 x 12345 + 6 =
9 x 1234 + 5 =
9 x 123 + 4 =
9 x 12 + 3 =
9 x 1 + 2 =

6)

8 x 123456789 + 9 =
8 x 12345678 + 8 =
8 x 1234567 + 7 =
8 x 123456 + 6 =
8 x 12345 + 5 =
8 x 1234 + 4 =
8 x 123 + 3 =
8 x 12 + 2 =
8 x 1 + 1 =

Digit Sum Patterns

N

The digit sum of a number is found by adding all the digits in the number until a single digit number results.

e.g.4361 has a digit sum of

$4+3+6+1 \rightarrow 14 \rightarrow 1+4 \rightarrow 5$

Find the digit sum for the multiples of two

		Digit Sum
1 x 2 =	2	2
2 x 2 =	4	4
3 x 2 =	6	6
4 x 2 =	....	....
5 x 2 =	10	1
6 x 2 =	12	3
7 x 2 =	....	....
8 x 2 =	....	....
9 x 2 =	....	....
10 x 2 =	....	....
11 x 2 =	....	....
12 x 2 =	....	....
13 x 2 =	....	....

Write down any patterns you notice. Predict the digit sums for the 14th, 15th, 16th, 17th and 18th multiples of two.

The pattern repeats every nine cycles and therefore has a period of 9. Find the period for the multiples of three, four, five, six, seven, eight and nine. Which multiples have similar periods?

Three Guzinta

N

- The numbers 21, 36, 93 and 300 are all divisible by three, that is when these numbers are divided by three, there are no remainders.
- Consider the digit sums of the above set of numbers

21 → 2 + 1 → ....

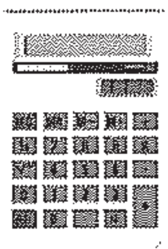
36 → 3 + 6 → ....

93 → 9 + 3 → 12 → 1 + 2 →.............

300 → 3 + 0 + 0 → ....

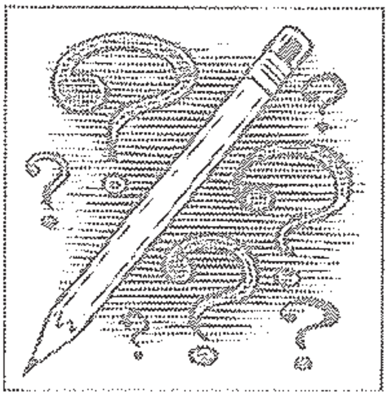
- Write down what you notice about these results.
- Predict which of these numbers would be divisible by three.

41, 57, 84, 92, 101, 102, 3664



- Check your predictions

- Try to write a rule to determine whether a number is divisible by three.



Four Guzinta

N

- ✱ List some two-digit multiples of 4,
  e.g. 16, 24, 40, 48, 80.
- ✱ Place up to six digits in front of any of these numbers,
  e.g. 4175**48**
- ✱ Divide your number by four.
  What do you notice?
- ✱ Try making some more numbers where the last two digits are divisible by four. Divide these numbers by four.
  What do you notice?
- ✱ Try to write a rule for testing whether a number is divisible by four.

Eight Guzinta

- ✱ List some three-digit multiples of 8,
  e.g. 120, 168, 256, 408, 888.
- ✱ Place up to five digits in front of any of these numbers,
  e.g. 615**168**
- ✱ Divide your number by eight.

What do you notice?

- ✱ Repeat several times.
- ✱ Try to write a rule to test whether a number is divisible by eight.

Nine Guzinta

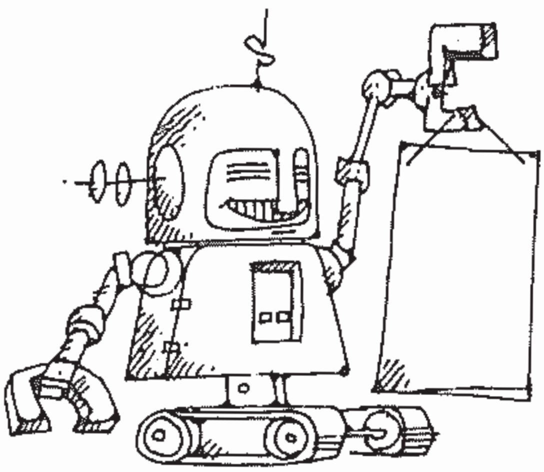
N

Try the following procedure.

- Choose a 3 digit number, e.g. **482**
- Find the digit sum, i.e. **4 + 8 + 2 → 14 → 5**
- Divide the original 3 digit number by 9 and find the remainder, e.g. **482 ÷ 9 =**

Try this procedure several times and write down what you notice about the digit sum and the remainder.

You probably noticed from the "Ninety Nine Times" activity that when you add the digits of a number that is divisible by nine (without leaving a remainder) – the digits add up to 9 or a multiple of nine.



Try to write a rule to predict whether a number is divisible by nine. Check your rule with a partner.

Try to find rules of divisibility for other numbers. For example enter 5 + = = = into your calculator to make your calculator produce the multiples of five. What do you notice about the units digit every time you press equals?
Try to write a rule to determine whether a number is divisible by five.

Eratosthenes' Sieve

N

- Cross out the number 1.
- Circle the number 2 and then cross out all the multiples of 2.
- Circle 3 and then cross out all the multiples of 3.
- Circle 5 and cross out all the multiples of 5.
- Circle 7 and cross out all the multiples of 7.

~~1~~	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

You should be left with the following numbers:
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, and 97.
What is special about this set of numbers?

Eratosthenes was a Greek mathematician who first devised a method for finding all the prime numbers less than 100.

Number Charts

N

Many interesting patterns may be found by examining a table chart.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

Consider any square within the table chart

e.g.

4	6
6	9

4 x 9 = 36 6 x 6 = 36.

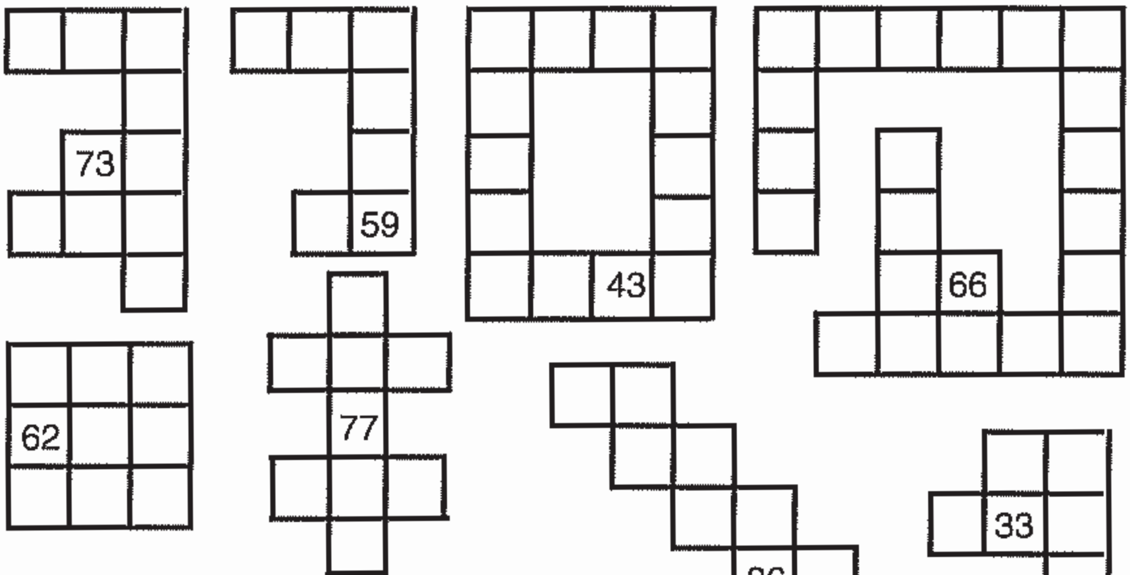
Is this an isolated occurrence? Try some other 2 x 2 squares.
What happens with larger squares? 3 x 3, 4 x 4 … 10 x 10?

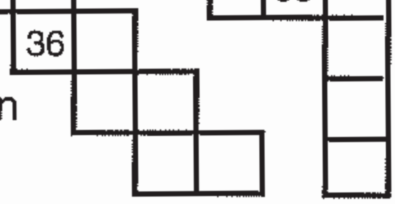
Try some of the other number charts in the appendix and see what happens.

Jumbled Charts

N

Without looking at the 1–100 chart, try to fill in the missing numbers



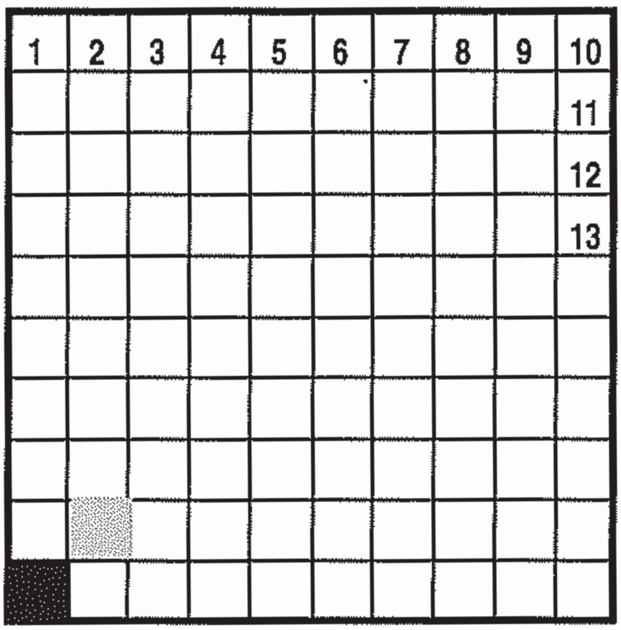


Try making some of your own.
Simply cut up a number chart (you can use 1–100, 0–99 or a table chart) and blank out all but one number.

Give them to your friends to try.

Number Spiral

Use a blank hundred chart and number it as shown (stop at 13).
Place a tick where you think 50 will end up and a cross where you think 100 will be. Without counting, predict what numbers will be in the black square and the shaded square.
Now complete the chart to verify your prediction

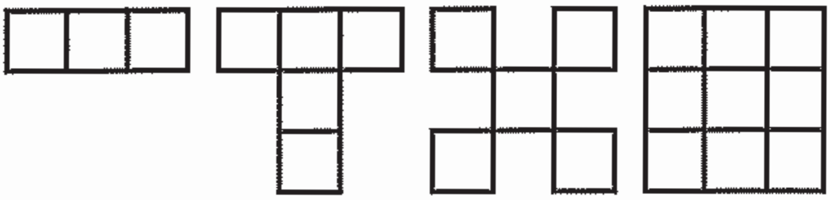


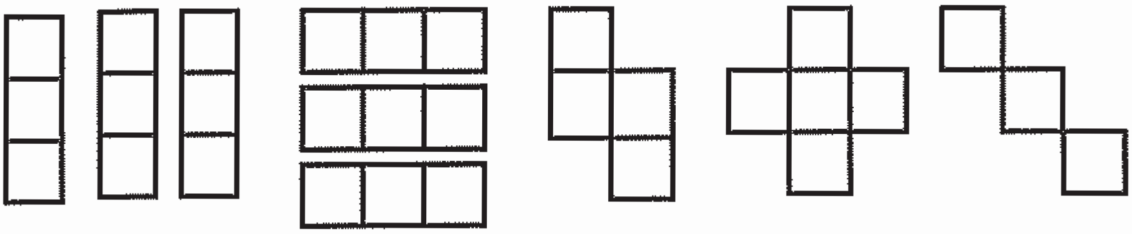
Windows

N

Choose a number chart (e.g. 0—99) and make two copies. Cut some windows out of one chart and then overlay it on top of the other chart.

Examples of windows:-





Look for patterns within each window.
Add all the numbers in the window and note how the totals change as you move horizontally, vertically and diagonally across the chart.
Try some other charts from the appendix. Can you find any multiplication patterns?

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Covering Patterns

N

- Make several copies of the 1 – 100 number chart shown in the appendix..
- Colouring in all the multiples of nine gives the following pattern:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- Try to describe the pattern. Why does this pattern occur?
- Colour in all the multiples of 11 and try to describe the pattern found. Why does this pattern occur?
- Now try shading in multiples of 2, 3, 4, 5, 6, 7, 8 and 12 on separate number squares. Try to describe the patterns formed.
- Do the same patterns form if you use a different number chart?
  Try some found in the appendix.

1 — 100 Chart Patterns

N

Patterns may be found by adding or multiplying numbers in the opposite corners of the shapes drawn on the hundreds chart.

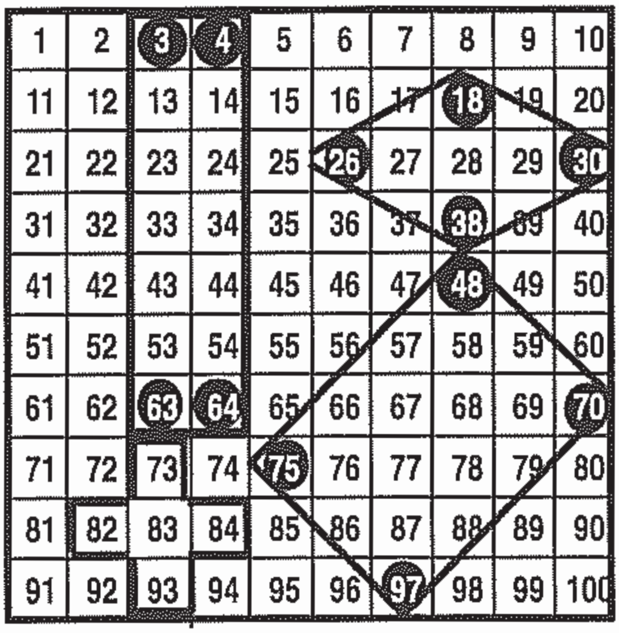
For example in the 3, 4, 64, 63 rectangle

$$3 + 64 = 4 + 63$$

$$3 \times 64 = 192 \text{ while } 4 \times 63 = 252$$

The difference between these two answers is 60. Is there a connection between the difference and the numbers in the rectangle?

What happens if you try other shapes? Investigate.



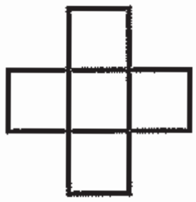
- Try drawing a cross  on the number chart.
- Try to find a relationship between the sum of the five numbers and the middle number?
- Try different size crosses and different number charts.

Table Chart Patterns

N

- Starting at the top left hand corner of your tables chart draw successively larger squares

1

1	2
2	4

1	2	3
2	4	6
3	6	9

1	2	3	4
2	4	6	8
3	6	9	12
4	8	12	16

Add the numbers in each square.

- Try to predict the sums for the next three square shapes. Check your predictions.

This set of numbers is called the square numbers.

Further patterns may be found by examining a tables chart. If we start at the top left hand corner of the tables chart and draw successively larger reverse ‘ L’ shapes a pattern is formed.

1

	2
2	4

		3
		6
3	6	9

			4
			8
			12
4	8	12	16

Add the numbers in each ‘ L’ shape.

- Try to predict the sums for the next 3 ‘ L’ shapes. Check your predictions.

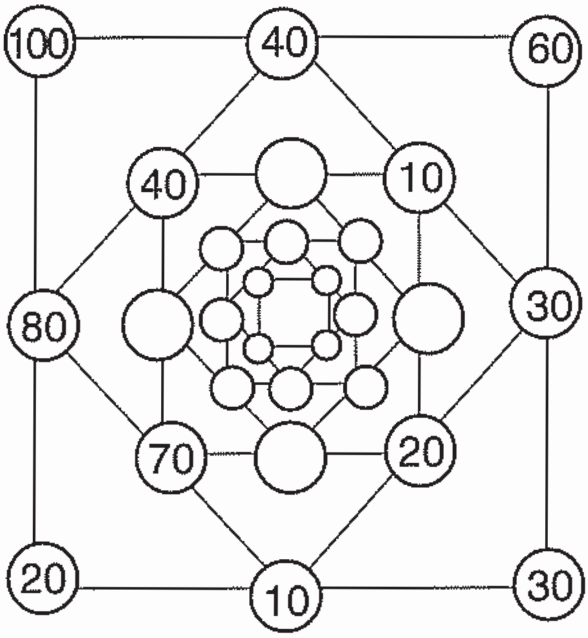
This set of numbers is called the cubic numbers.

NESTED SQUARES

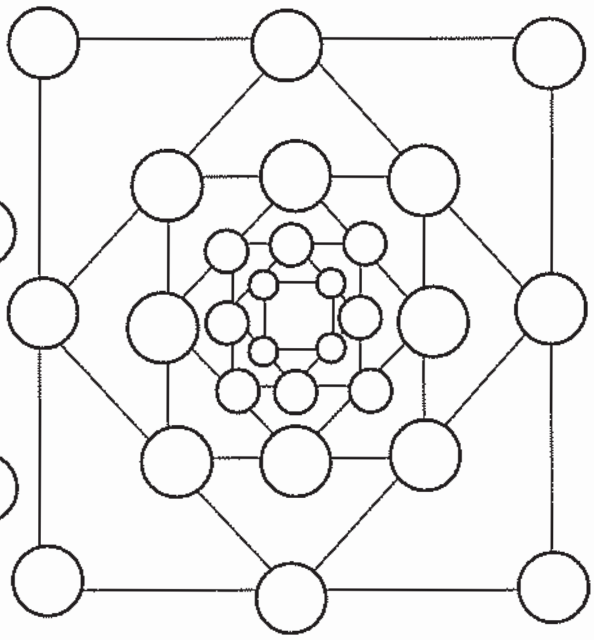
N

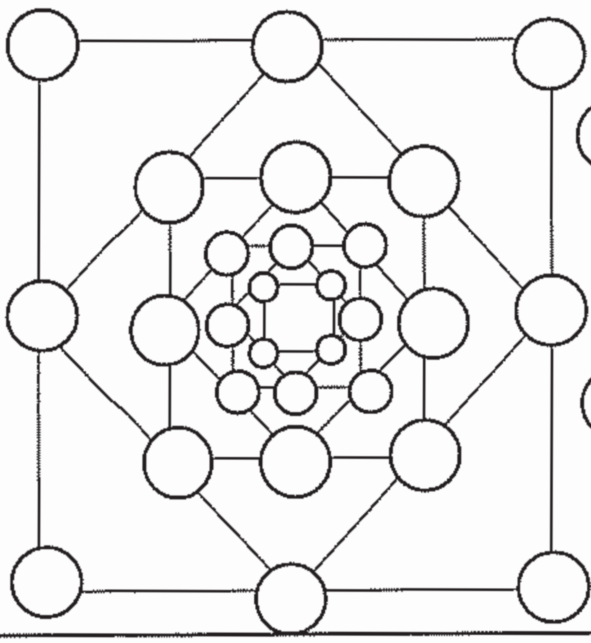
Place any four numbers on the outside corners.
Work out the difference between the numbers and write this in the circles in between.
Continue as far as you can go.
What do you notice?

The first one has been started for you.



Try using your own starting numbers.

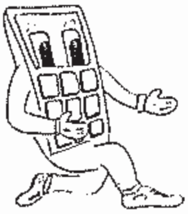




PUZZLING PATTERNS

N

Complete the first three multiplications and then try to predict the next numbers in the sequence.



91 x 11 =
91 x 111 =
91 x 1 111 =
91 x 11 111 = ???
91 x 111 111 = ???

3 x 12 x 34 =
3 x 13 x 34 =
3 x 14 x 34 =
3 x 15 x 34 = ???
3 x 16 x 34 = ???

9 x 1 − 1 =
9 x 21 − 1 =
9 x 321 − 1 =
9 x 4321 − 1 = ???
9 x 54 321 − 1 = ???

37 037 x 3 =
37 037 x 6 =
37 037 x 9 =
37 037 x 12 = ???
37 037 x 15 = ???

2 222 222 x 9 =
3 333 333 x 9 =
4 444 444 x 9 =
5 555 555 x 9 = ???
6 666 666 x 9 = ???

3367 x 33 =
3367 x 66 =
3367 x 99 =
3367 x 132 = ???
3367 x 165 = ???

199 x 11 =
299 x 11 =
399 x 11 =
499 x 11 = ???
599 x 11 = ???

6 x 7 =
66 x 67 =
666 x 667 =
6666 x 6667 = ???
66666 x 66667 = ???

9109 x 1 =
9109 x 2 =
9109 x 3 =
9109 x 4 = ???
9109 x 5 = ???

99 x 12 =
99 x 23 =
99 x 34 =
99 x 45 = ???
99 x 56 = ???

APPENDIX

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Solutions

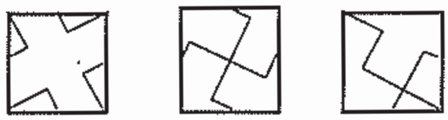
Finding Squares

The area of each subsequent square is half that of the previous square.

Square Trays

The largest volume (588 cm^3) is achieved when a 30mm square is cut from each corner

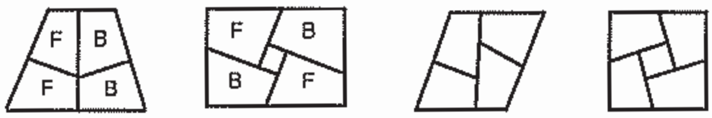
Squares from Crosses



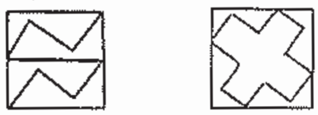
Hexagons into Star

Stars into Hexagon

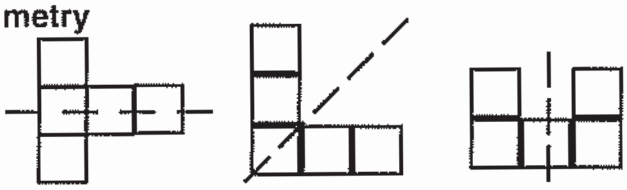
Cutting and Shaping



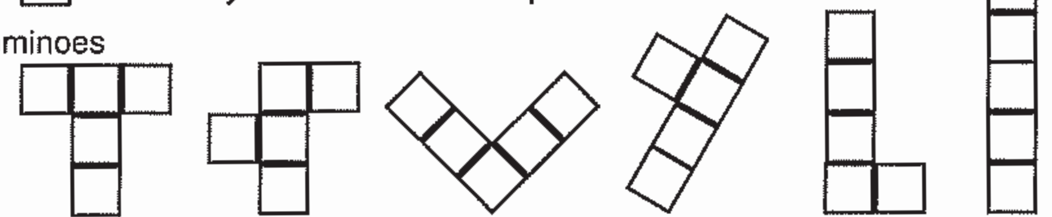
Cross Cut



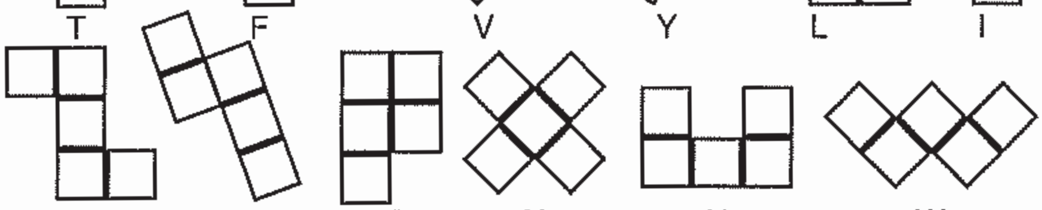
Symmetry



Pentominoes



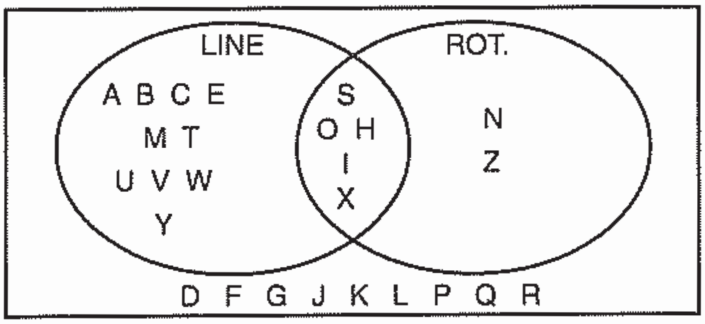
T F V Y L I



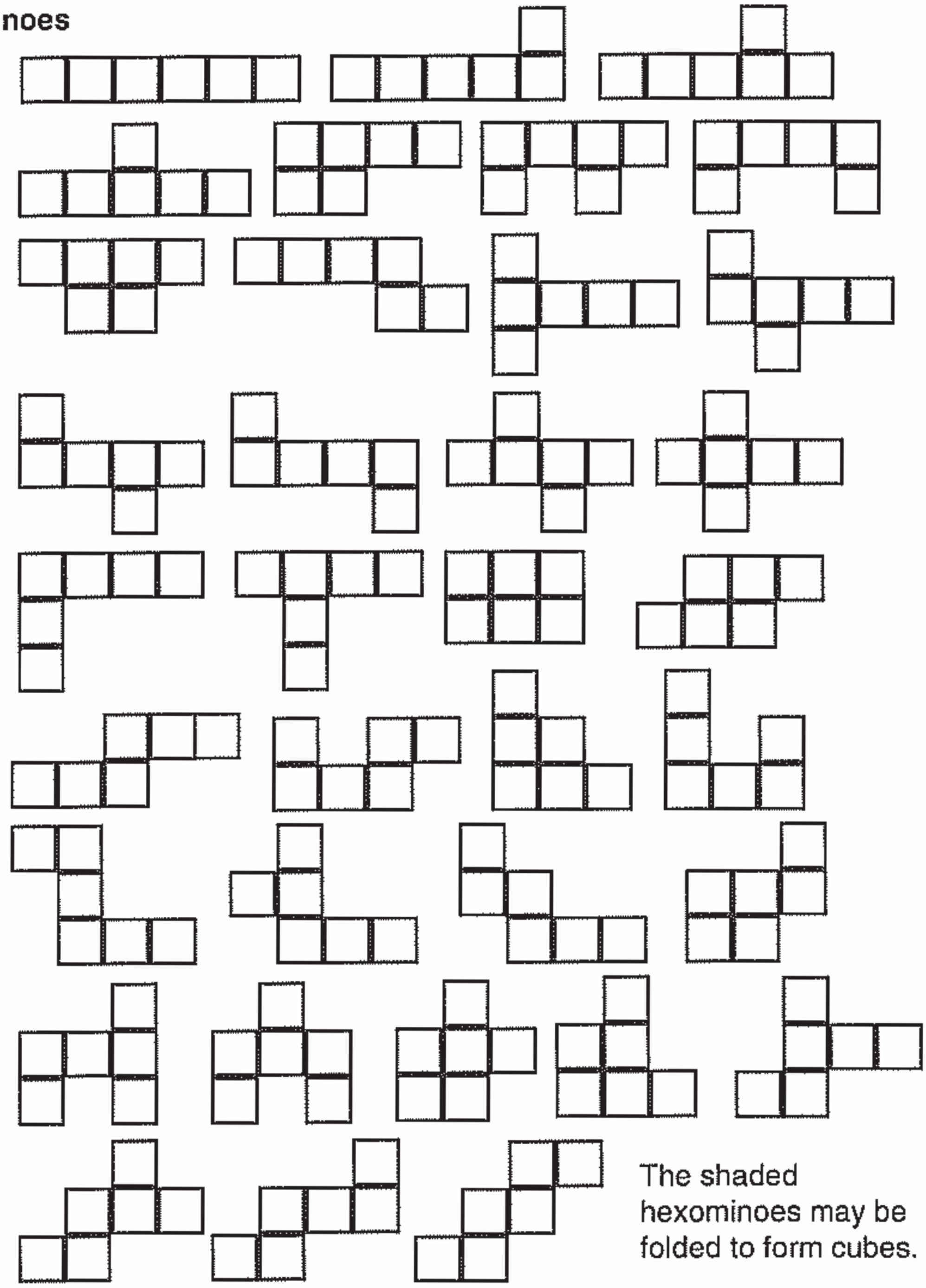
Z N P X U W

These pentominoes may be folded to form boxes without lids: L, X F, T Y, Z, N and W

Letters of the Alphabet



Hexominoes



The shaded hexominoes may be folded to form cubes.

Mathematical Tricks I

If n, n + 1 represent the two consecutive numbers, then the instructions give 2n + 1 2n + 10 n + 5 5 so it can be shown that we are always left with 5 as an answer.

The number is repeated twice. This occurs because multiplying by 7 and then by 1443 is the same as multiplying by 10101.

Mathematical Tricks II

If the two numbers are a, b the instructions give:

5a 5a + 7 10a + 14 10a + b + 14 10a + b

The first number appears in the tens place and the second number in the ones place

To Guess a Person's Age

If the three digits are a, b, c, then taking in to account place value, one value that might be formed is 100a + 10b + c (there are several other possibilities). The digits may be rearranged to form many different numbers, for example 100c + 10b + a, where the digits are simply reversed.

Subtracting the smaller from the larger:

100a + 10b + c – (100c + 10b + a) gives 99a – 99c or 99 (a – c).

A factor of 99 occurs for all the various combinations that may be produced with the three digits. Multiplying any number by 99 produces a digit sum that adds to 9, hence the instruction to find the digit sum and then add nines until a reasonable age is reached.

Mathematical Tricks III

Let m represent the month and a the age

The steps produce m 2m 2m + 5 100m + 250 (Note this step moves the month into the hundreds place.

100m + a + 250 100m + a – 115 Adding 115 gives 100m + a.

The month appears first, the age follows.

Mathematical Tricks IV

This trick is similar to the previous trick except that 365 is added rather than subtracted, hence the need to subtract 615 at the end to reveal the house number and age.

Table Square Sums

21, 42, 84 126 441 (which is the sum of the first row squared, i.e. 21 x 21)

The sum of all the numbers in a 4 x 4 square is $(1 + 2 + 3 + 4)^2$ i.e. 100

An 8 x 8 square would give $(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8)^2$ or 1296.

A 10 x 10 square would give $(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10)^2$ or 2704

The Ninety Nine Times Table

9, 18, 27, 36, 45, 54, 63, 72, 81. The units digit goes down by one while the tens digit goes up by one. The digits add to nine.

If the digits add to nine or a multiple of nine, then the number will divide by nine without any remainder.

34 x 9 = 306 3 + 0 + 6 = 9

Yes the pattern continues for larger multiples of nine.

99,198, 297, 396, 495, 594, 693, 782, 891. Yes the digits all add to eighteen and one and eight add to nine.

Tables Test 8 and 9

1)	1 111 111 111	2)	111 111 111
3)	987 654 321	4)	98 765 432
5)	1 111 111 111	6)	987 654 321
	111 111 111		98 765 432
	11 111 111		9 876 543
	1 111 111		987 654
	111 111		98 765
	11 111		9 876
	1 111		987
	111		98
	11		9

Digit Sum Patterns

2, 4, 6, 8, 1, 3, 5, 7, 9, 2, 4, 6, 8. The digit sums cycle through the even numbers and then all the odd numbers. 1, 3, 5, 7 and 9.

Multiples of 3 have a period of three. The cycle 3, 6, 9 is repeated.

The multiples of six have a similar period, but the cycle is 6, 3, 9.

The multiples of 4 have a digit sum period of nine. The cycle 4, 8, 3, 7, 2, 6, 1, 5, 9 is repeated.

The multiples of 5 also have a period of nine. The cycle 5, 1, 6, 2, 7, 3, 8, 4 and 9 is repeated.

The digit sum for *the multiples of 7* repeats every nine digits. The digits 7, 5, 3, 1, 8, 6, 4, 2, and 9 continue to cycle through. The period for the multiples of 7 and 2 digit sums are the same.

The multiples of 8 produce an interesting digit sum pattern of 8, 7, 6, 5, 4, 3, 2, 1 and 9 before the cycle repeats. The period therefore is nine.

Three Guzinta

3, 9, 3, 3. The digit sums are all multiples of three.

The following would be divisible by three without leaving a remainder. 57, 84, and 102.

A number will be divisible by three without leaving a remainder if all the digits of the number add to 3, 6 or 9.

Four Guzinta

The new numbers are also divisible by four without leaving any remainders.

A number is divisible by four if the last two digits of the number divide by four without a remainder.

Eight Guzinta

The number that is formed is divisible by eight without leaving a remainder.

A number is divisible by eight if the last three digits of the number divide by eight without leaving a remainder.

Nine Guzinta

The remainder is equal to the digit sum.

A number will be divisible by nine without leaving a remainder if the digit sum adds to nine.

A number will be divisible by five without leaving a remainder if the units digit is 5 or 0.

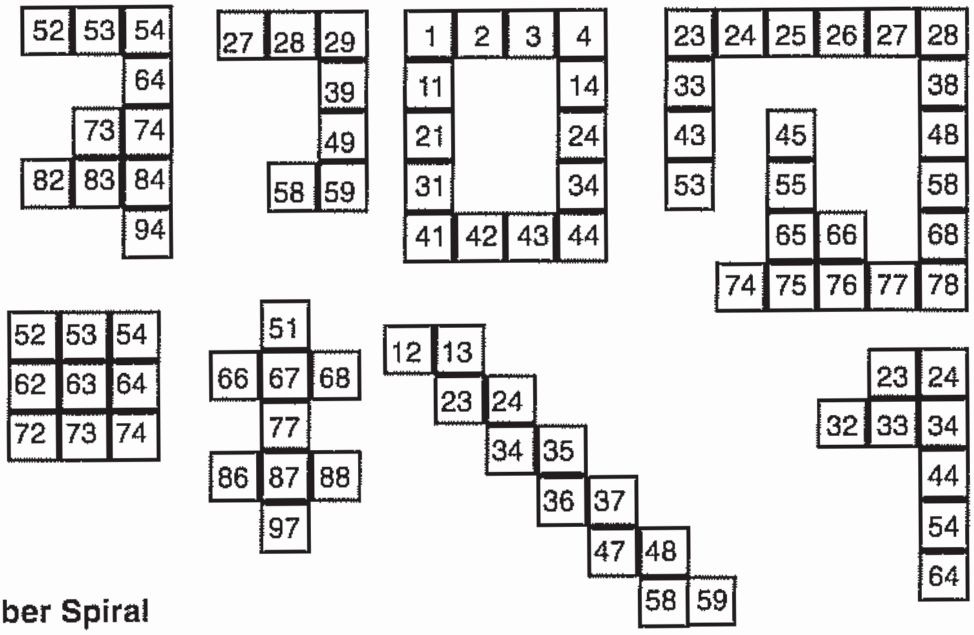
Erosthene's Sieve

The remaining numbers are all prime numbers.

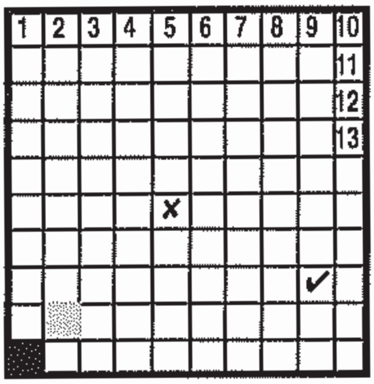
Number Charts

No. This is not an isolated occurrence. The same pattern occurs in larger squares.

Jumbled Charts



Number Spiral



Windows

Answers will vary depending on window used.

Covering patterns

The multiples of nine form a diagonal that runs from the top right to the bottom left of the chart. Nine is one less than ten. Eighteen is two less than twenty and so on.

The multiples of eleven form a diagonal starting from the top left to the bottom right of the chart. Eleven is one more than ten. Twenty two is two more than twenty and so on.

Multiples of 2, 3, 4 and 5 form columns.

Multiples of six form four diagonals.

No real pattern is formed by multiples of seven.

The three diagonals from right to left are formed by the multiples of eight.

Six different patterns are formed if different number charts are used.

1–100 Chart Patterns

Yes there is a connection. A difference of 60 exists between the starting numbers (63 – 3, 64 – 4) and the same difference occurs when the numbers in opposite corners are multiplied. This may be illustrated by the distributed property

(3 x 60) + (3 x 4)

(4 x 60) + (4 x 3)

A variety of patterns occur when using other shapes such as the parallelogram.

The cross produces an interesting relationship. Multiplying the centre number of the cross by five gives the same result as adding the five numbers contained in the cross.

Table Chart Patterns

The sequence 1(1^2) 9(3^2) 36 (6^2) 100 (10^2) is produced by adding values in each of the squares. The sums for the next three would be 15^2 or 225 21^2 or 441 and 28^2 or 784.

The'" L" shapes form the following sequence 1(1^0) 8(2^3) 27(3^3) 64(4^3) The sum for the next three "L" shapes would be 5^3 or 125, 6^3 or 216 and 7^3 or 343

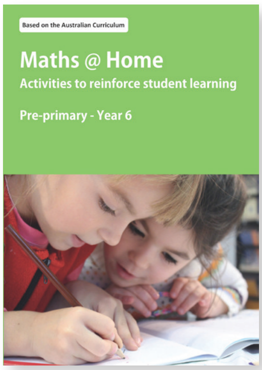
Nested Squares

The square continues until you reach zero

Puzzling Patterns

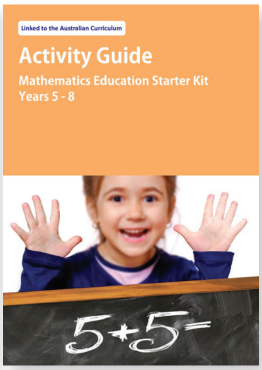
1001, 10 101, 101 101, **1 011 101, 10 111 101**.
1224, 1326, 1428 **1530, 1632.**
8, 188, 2888, **38 888, 488 888.**
111 111, 222 222, 333 333, **444 444, 555 555.**
19 999 998, 29 999 997, 39 999 996, **49 999 995, 59 999 994.**
111 111, 222 222, 333, 333, **444 444, 555 555.**
2189, 3289, 4389, **5489, 6589.**
42, 4422, 444 222, **4 444 222, 4 444 422 222.**
9109, 18 218, 27 327, 36 436, 45 545.
1188, 2277, 3366, **4455, 5544 .**

More MAWA resources and student activities may be found at mawainc.org.au including:

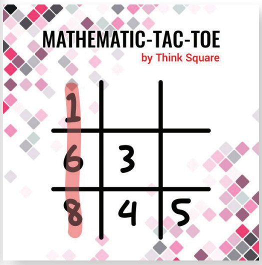


Maths @ Home

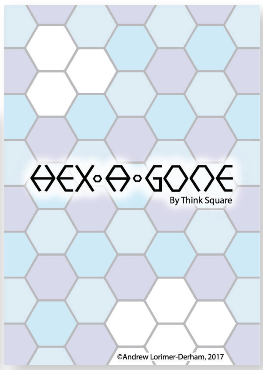
Mathematics Education Starter Kit



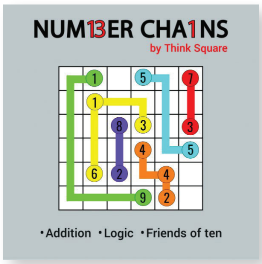
Activity Guide



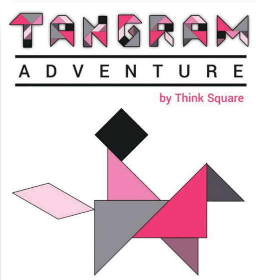
Mathematics Tic-Tac-Toe



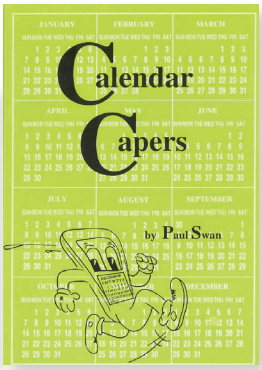
Hexagone



Number Chains



Tangram Adventure Puzzle Book



Calendar Capers

Countdown Wiz

Take Sum Risks

More Dr Paul Swan titles may be found at drpaulswan.com.au